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Structure and reactions of halo nuclei: An entangled approach

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Abstract. Halo nuclei are characterised by their weak binding, large spatial extent and hence a quite pronounced, yet highly correlated, few-body structure. This is typically in terms of a well-defined core plus one or more valence nucleons. Over the past decade the properties of halo nuclei have been studied theoretically using a range of reaction models, many of which having served us well for half a century or more in the study of less exotic, "mean-field", nuclei. However, it is now clear that for many reactions with halo nuclei, it is not appropriate to disentangle (factorise out) the structure information from the reaction information. That is, the few-body nature of these systems requires few-body reaction models in which the nuclear structure and reaction mechanisms are necessarily entangled. This talk will briefly review the physical assumptions made by various reaction models, and point to areas where progress is being made to extend their range of applicability in order to provide further insights into halo structure.

PACS. 21.45.+v Few-body systems – 24.10.-i Nuclear reaction models and methods – 24.50.+g Direct reactions – 25.10.+s Nuclear reactions involving few-nucleon systems

1 Introduction

The current wave of interest in the study of the properties of light dripline nuclei, in particular the nuclear-halo phenomenon, was triggered by a series of interaction cross-section measurements carried out by Tanihata and co-workers at Berkeley in 1985 [1]. The suggestion that the large matter radii of neutron-rich nuclei such as ^{11}Li are due to the very weak binding of the last two valence neutrons was made soon after by Hansen and Jonson [2]. Since then, a number of other nuclei (such as ^{6}He, ^{8}He, ^{11}Be, ^{14}Be, ^{15}C and ^{19}C) have been shown to exhibit neutron halos (see the reviews in [3–6]). Due to their very short lifetimes, such nuclei must constitute a beam incident on a stable nuclear target. Most experiments to date have involved the production of these exotic species via the fragmentation of a primary beam of stable nuclei on a production target. The secondary beam of exotic nuclei is then directed onto a secondary target and a variety of relatively high-energy reactions are studied to probe their properties.

Many of these reactions have provided an exciting challenge to theorists interested in both the nuclear structure of these exotic systems as well as the reaction mechanisms involved in their scattering and breakup on a variety of targets and over a range of energies. In this paper I will present a short and selective review of the status of various reaction models that are currently being developed and used to study the properties of halo nuclei.

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2 Few-body structure models

The most important, and remarkable, feature of the nuclear halo follows from basic quantum mechanics. Due to the very weak binding of the last one or two valence nucleons to the rest of the nucleus, the wave function describing their relative motion has a spatial distribution that extends far beyond the range of the attractive nuclear binding potential. This highly open (cluster) structure can largely be accounted for by modeling these nuclei as few-body systems. Indeed, more traditional methods of describing their structure within the shell model struggle to reproduce basic properties, such as their large matter radii. Such models, while working very well for mean-field nuclei, do not contain the important correlations arising from the few-body degrees of freedom. In particular, most reactions with halo nuclei tend to involve processes that are highly surface dominated; thus the asymptotic behaviour of the nuclear wave functions that follows from their few-body nature is crucial.

Halo nuclei are so weakly held together that they typically have just one bound state, in which the one or two valence nucleons are in a low relative angular-momentum state ($\ell = 0, 1$) with respect to the rest of the nucleons that make the more strongly bound core. Higher relative orbital angular-momentum states give rise to large centrifugal barriers that tend to hold the valence nucleons more closely to the core. The simple picture that lends itself to such a structure is that of a cluster system of a structureless core (c) plus valence nucleons. Of course, projecting the many-body wave function for the full A-nucleon system onto such two- or three-body model spaces means that the resulting wave functions are not fully antisymmetrised, and a number of studies have been carried out to investigate possible Pauli-blocking mechanisms to address this problem (*e.g.*, [7]).

In this paper I will treat one- and two-nucleon halos separately due to the huge increase in complexity in going from a two-body projectile (requiring a three-body reaction model) to a three-body projectile (four-body reaction model). I will also restrict myself to discussing reactions with neutron halo nuclei, with the exception of ^{8}B (treated as $^7\mathrm{Be} + \mathrm{proton}$). This nucleus has such a small separation energy (137 keV) that it qualifies as the best candidate for a proton halo despite the Coulomb repulsion between the core and the proton and their relative $\ell = 1$ angular-momentum state.

One-neutron halo nuclei such as ^{11}Be can be modeled therefore as two-body systems ($c+n$) bound by a potential with parameters chosen to give the correct binding energy and rms matter radius. The structure of such nuclei has much in common with that of the deuteron. Indeed the deuteron is often referred to as the simplest halo nucleus. (One is free of course to consider it as either a neutron or proton halo nucleus!) What is of relevance here is that many of the 3-body models developed originally to describe reactions involving the deuteron, and indeed ^{6}Li (treated as an $\alpha + d$ two-cluster system), are now being applied to study one-neutron halo nuclei such as ^{11}Be.

Two-neutron halo nuclei such as ^{6}He, ^{11}Li and ^{14}Be are more interesting and challenging from a theoretical point of view. These are the so-called Borromean nuclei [8] which, within a three-body ($c + N + N$) model, have no two-body subsystem bound states. Here too, the core degrees of freedom are decoupled from the relative degrees of freedom between the core and the valence particles, and the many-body wave function is approximated by

$$\Phi(1, 2, \ldots, A) = \varphi_c(\xi)\, \psi^{(3)}_{JM,T}(1, 2) \;, \qquad (1)$$

where $\psi^{(3)}_{JM,T}$ is the three-body wave function of relative motion. A number of theoretical approaches have been employed to solve this three-body problem [8–13] and will not be discussed here. What is important is that the three-body asymptotics need to be accounted for correctly in the wave function if we are to have any hope of extracting useful information from reactions. In addition, the few-body correlations that are built into these structure models must be retained in any reaction models.

3 Reaction models

An important consideration in the study of reactions with halo nuclei is that they are easily broken up in the nuclear and Coulomb fields of the target nucleus. Therefore, excitations of the halo nucleus into the continuum (since they typically have only one bound state) must be included in the reaction model. Intermediate-state coupling to the continuum rules out "one-step" models such as DWBA for most reactions of interest.

3.1 High-energy reactions

Most of the experiments with halo nuclei have been at energies high enough for a number of simplifying assumptions to be made to the reaction models, which make them both tractable and transparent. I will therefore focus mainly on such fragmentation reactions in which the projectile is incident at energies above 50 MeV/A, and then discuss what options are available if we wish to reliably model reactions at lower energies. The reactions I will discuss in this paper will be classified according to both the energy regime, and whether the projectile is a one- or two-neutron halo nucleus.

3.1.1 Reaction cross-sections and halo sizes

As a preliminary example of the importance of the few-body degrees of freedom of halo nuclei in reaction calculations I mention their application in total reaction cross-section studies to deduce the root mean-square matter radii. Figure 1 shows a plot of the rms radii for a range of lithium isotopes predicted from a comparison [14] of calculated reaction cross-sections with measured interaction cross-sections.

Early estimates of the size of ^{11}Li employed a model in which the matter distribution of the halo nucleus was described by a one-body density [15]. This amounts to

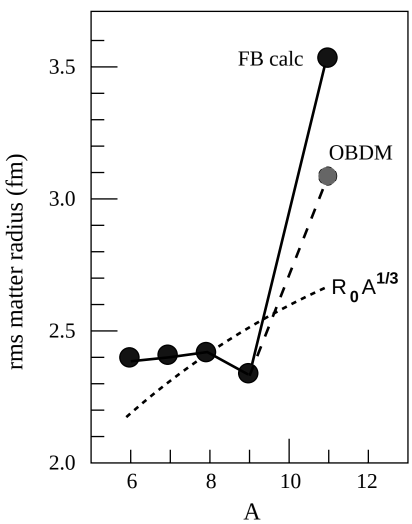

Fig. 1. Predicted rms matter radii of the lithium isotopes from an optical limit Glauber calculation using a one-body density for the projectile [15], and from a few-body Glauber model [14]. The solid and dashed lines are to guide the eye.

taking the optical limit [16] of the Glauber model [17], whereby the projectile's few-body degrees of freedom are integrated over *before* the scattering is calculated. This predicted an enhanced size compared with one obtained from the usual $\langle r^2\rangle^{1/2} \propto A^{1/3}$ scaling.

By retaining the few-body degrees of freedom in the ^{11}Li wave function, this structure information remains entangled in the reaction formalism. Now, however, an even larger matter radius was deduced. This may at first sight seem contrary to what we might expect, since such a model allows for new breakup channels to become available and hence it might be expected that a larger reaction cross-section would be predicted. Subsequently, a smaller radius would be required to bring the cross-section back down again. A simple theoretical proof, due to Johnson and Goebel [18], shows that for a given halo wave function, the folding model (static density limit) always *overestimates* the total reaction cross-section for strongly absorbed particles, thus requiring a smaller radius.

3.1.2 Reaction models for one-neutron halos

Three-body reaction models ($c + n + T$) involving one-neutron halo nuclei treat only the halo degrees of freedom explicitly, while the core and target excitations can be included through appropriate complex optical potentials to describe the separate $c+T$ and $n+T$ scattering. The starting point for all such models is the three-body Hamiltonian in the c.m. frame

$$H = T_R + H_0 + V_{cT} + V_{nT}\,, \tag{2}$$

where $H_0 = T_r + V_{cn}$ is the internal Hamiltonian of the projectile whose eigenstates satisfy

$$\begin{aligned} H_0\psi_i(\mathbf{r}) &= \epsilon_i\psi_i(\mathbf{r})\,, \qquad i = 0,1,2,\ldots,\\ H_0\psi_{\mathbf{k}}(\mathbf{r}) &= \epsilon_k\psi_{\mathbf{k}}(\mathbf{r})\,, \qquad \epsilon_k = \frac{\hbar^2k^2}{2\mu_{cn}}\,, \end{aligned} \tag{3}$$

where ψ_0 is the projectile ground state, with c-n separation energy ϵ_0, and $\psi_{\mathbf{k}}$ denote the projectile breakup (or continuum) states. The scattering wave function satisfying $H\Psi = E\Psi$ requires both the core and target nuclei to be left in their ground states after the reaction, but it includes components in which the projectile is broken up into its two constituents. The complete problem is solved by expanding Ψ in the set of bound and continuum states of the $c+n$ system. If coupling to the continuum is ignored then the problem can be written as a set of coupled equations between the discrete bound states. However, for halo nuclei, as with the deuteron, there are typically only one or two bound states, close to the breakup threshold, and thus coupling to the continuum, and indeed between continuum states, is very important. The most precise method of dealing with this problem is to map the continuum onto a discrete square-integrable basis that is orthogonal to the bound states. This amounts to "chopping up" the continuum into energy bins that act as effective discrete excited states of the projectile and allows the problem to be solved within a coupled-channels approach. This is the so-called coupled discretised continuum channels (CDCC) method [19,20], and is the most accurate reaction model currently available for one-nucleon halo nuclei.

At high incident energies, however, a number of different approximation schemes can be reliably applied. These not only make the problem far easier to solve, but offer useful insights into the different mechanisms involved in specific reactions.

One of the most common approaches is to make use of the adiabatic, or "sudden", approximation [21] whereby it is assumed that the interaction time between the projectile and target is sufficiently short that the halo degrees of freedom can be regarded as frozen. Thus, the projectile's internal Hamiltonian H_0 is replaced in eq. (2) by its ground-state binding energy, ϵ_0. In this way, the few-body Schrödinger equation contains only parametric dependence on the variable $\mathbf{r}$ through V_{cT} and V_{nT}. The transition amplitude to a given final state of the projectile is then found by projecting the "adiabatic" wave function onto that state.

A special case of the adiabatic model can be applied when V_{nT} is small or zero (such as in Coulomb breakup). This is known as the "recoil limit" model [22] since the halo can only be broken up via recoil of the core. In this model, the elastic amplitude factorises into a point amplitude that, to a good approximation, is that of the core-target system at the same energy per nucleon and momentum transfer as the original projectile, and a form factor containing all the information on the structure and excitation of the halo nucleus. Such a simple model may not always be very precise (such as in nuclear breakup when V_{nT} is not zero) but it does provide a clear indication of the importance of coupling to the breakup channels when dealing with halo nuclei.

The most successful few-body approach for calculating probabilities and cross-sections for a range of reactions involving halo nuclei has been based on Glauber's multiple scattering diffraction theory for composite systems [17, 23]. The model requires making, in addition to the sudden approximation, an eikonal (straight-line trajectory) approximation. As discussed in subsubsect. 3.1.1, further approximations leading to the optical limit of the model are not reliable for most reactions with halo nuclei since they neglect important few-body correlations.

The Glauber few-body model belongs to a more general theoretical framework based on the semiclassical approach whereby each of the constituents of the model few-body projectile travels along a definite trajectory defined by an impact parameter. Indeed, impact parameter methods have a history of being applied to reactions of loosely bound nuclei that predates the work of Glauber. For instance, stripping processes, such as in deuteron-induced reactions, have been studied using approaches developed by Serber [24]. These simple geometric methods have more recently been used to describe the narrow momentum distributions of the outgoing fragments following the breakup of halo nuclei. In fact, the general features of the distributions can be reproduced even with simple analytical models that treat the target as a black disk (strong absorption limit) in which the individual S-matrices for the projectile constituent-target interactions are treated as simple step functions [25]. More recently, reaction mechanisms have been included more correctly by using realistic interactions. While on the topic of momentum distributions, it is worth mentioning that the CDCC method has been used successfully to account for and explain effects that other approximation schemes cannot. For instance, it has been used [26] to calculate the parallel momentum distributions of the ground-state core fragments from the breakup of ^{11}Be and ^{15}C and accounts for a significant asymmetry and a low-momentum tail in the distribution which cannot be reproduced using eikonal models.

A number of other semiclassical three-body reaction models have been developed in recent years and applied to reactions in which the projectile is treated as a core + valence nucleon system. One method is to solve the time-dependent Schrödinger equation after assuming that the relative motion between the projectile's core and the target can be treated classically and approximated by a constant velocity path. This method [27,28] treats the time dependence of the reaction explicitly and thus conserves energy, but not momentum. Breakup amplitudes can then be calculated within time-dependent perturbation theory.

Another time-dependent approach [29], also treating the projectile-target relative motion semiclassically, is to solve the time-dependent Schrödinger equation using a non-perturbative algorithm on a three-dimensional spatial mesh that allows the treatment of Coulomb breakup in the non-perturbative regime.

A new approach, which has been named the uncorrelated scattering approximation (USA) [30], assumes that correlations between the projectile constituents can be neglected inside a region where they interact strongly with the target. Such an approach is valid for loosely bound projectiles and has so far been applied to elastic, inelastic and breakup reactions of the deuteron as a test case.

3.1.3 Reaction models for two-neutron halos

In order to treat reactions involving two-neutron halo projectiles, we require 4-body $(c+n+n+T)$ reaction models (see fig. 2). To date, a fully quantum-mechanical 4-body model, based on the CDCC method, does not exist and we must therefore rely for the moment on approximation schemes to deal with the problem. The most successful of these are based on Glauber methods. The first application of a four-body Glauber model to the scattering and breakup of halo nuclei was first made in the early 1990s [31]. A similar approach was adopted in [32]. In both these calculations however, the few-body correlations in the projectile are neglected and the halo wave function is approximated by a product of uncorrelated single-particle wave functions for the two halo neutrons. A more complete calculation, applied to elastic scattering [33] and total reaction cross-sections [34] was performed in which these important correlations were taken into account.

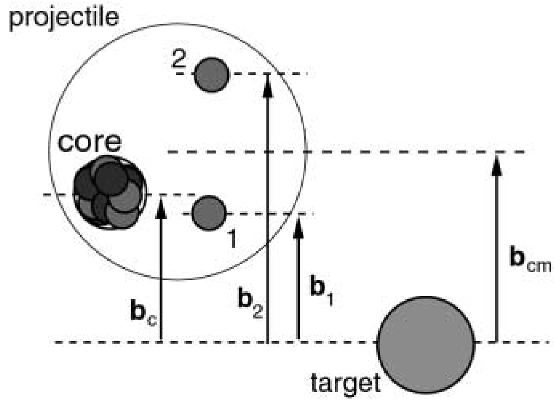

Fig. 2. Four-body reaction models of the scattering and breakup of two-neutron halo nuclei at high energy usually involve a semiclassical approach in which each of the three projectile constituents is assumed to move along a definite trajectory defined by its impact parameter.

A similar semiclassical approach that has been applied to breakup reactions of two-neutron halos, and which like the above Glauber methods assumes the projectile constituents travel along definite trajectories defined by impact parameters, is known as the participant/spectator model [35]. Unlike the Glauber approach, it does not make any eikonal assumptions about the trajectories, however it assumes that the reaction is a superposition of three independent processes in which each of the projectile's constituents interacts with the target while the other two remain as spectators.

The expressions for various integrated cross-sections, such as the total reaction cross-section and the stripping and diffractive dissociation cross-sections, are very simple within the Glauber model (see, for instance, [36,23] and references therein). More recently, interest has grown in the spectroscopic information that can be extracted from neutron knockout reactions in which the surviving fragment is detected in a definite angular-momentum state. A model based on the spectator core assumption [37] has been developed [38]. A 4-body stripping model, which goes beyond the spectator core approximation, has recently been developed [39]. Here, off-diagonal elements in the transition matrix of the surviving two-body $(c+n)$ fragment, after the removal of a valence neutron, are no longer neglected, as they are in the spectator core model. Such terms couple different initial and final single-particle states of this $c+n$ subsystem of the projectile. Corrections of the order of 10% to the total stripping cross-sections are found when these dynamical effects are included. Collective excitations of the surviving fragment are also expected to play a minor role.

An important common feature of the above four-body reaction models is that they all make the frozen halo (sudden/adiabatic) approximation. A full quantum-mechanical (in the sense of not making any semiclassical approximations) four-body adiabatic model has been developed [40] and applied to the elastic scattering of ^{11}Li. However, provided the energy is high enough for the eikonal assumption to be valid, then Glauber methods tend to be far more amenable numerically. In addition, due to the implicit closure assumption over the continuum of intermediate breakup states that is made in the few-body Glauber model, there are no convergence issues to worry about, unlike in the coupled-channels formulation of the adiabatic model.

A four-body model based on the distorted waves impulse approximation (DWIA) has been used [41] to study the low-energy continuum of two-neutron halo (Borromean) nuclei through elastic and inelastic breakup reactions on both low- and high-Z targets. This approach treats the three-body correlated structure of the projectile in both the initial and final states consistently, however it necessarily assumes that a one-step breakup process dominates. In the spirit of this review, it must be emphasised that such an assumption may not be valid since, as has been stated earlier, higher-order couplings to the continuum, and indeed within the continuum, are important when dealing with halo nuclei. Such considerations have been investigated for the case of breakup of one-nucleon halos [42,43] and, earlier, for elastic scattering of two-neutron halos [44].

Finally, it is worth mentioning that a 6-body Glauber model has been used to calculate the elastic scattering of ^{8}He (treated as a 5-body, $\alpha + 4n$, system) from a ^{12}C target [45] and from protons [46]. While ^{8}He can be considered as a four-neutron halo nucleus, its ground-state wave function does not have such a long range tail as ^{6}He. The elastic amplitude can be reduced to a 12-dimensional integration that must be solved numerically, using a Monte Carlo sampling method.

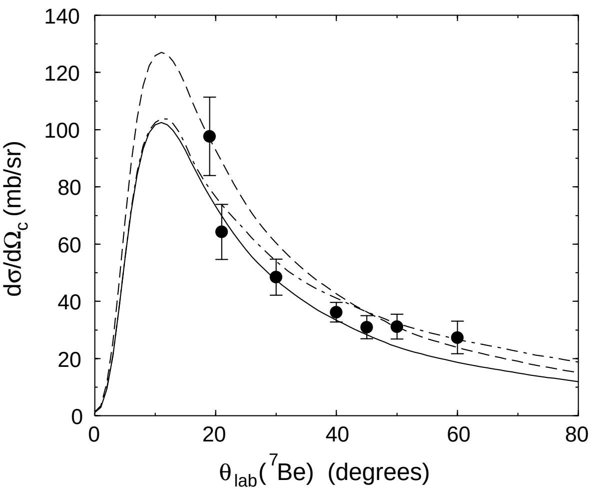

Fig. 3. Calculated cross-sections for ^{7}Be from the breakup of ^{8}B on a ^{58}Ni target at 25.8 MeV using the CDCC method. The various curves show the effects of different proton-^{7}Be and proton-target interaction potentials (described in ref. [43]. The experimental data are from ref. [47].

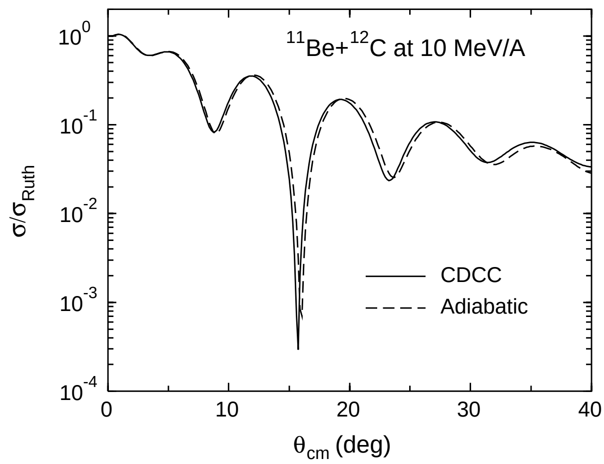

Fig. 4. ^{11}Be + ^{12}C elastic scattering cross-section at 10 MeV/A calculated using two 3-body models. The solid curve is from a full CDCC calculation, and the dashed curve assumes a sudden (frozen halo) approximation.

4 Extension to low energies

Few-body reactions at lower incident energies are far more difficult to treat consistently. Not only do nuclear and Coulomb interactions need to be treated within the same model to account correctly for interference effects, but multistep processes involving coupling to the continuum are even more important than at higher energies. An advantage of the CDCC method is its applicability at low energies where approximation schemes used in many other few-body approaches break down. Figure 3 shows the calculated breakup cross-section for ^{8}B + ^{58}Ni at just a few MeV per nucleon. However, as mentioned in the last section, the CDCC method can only be applied to reactions involving one-nucleon halo nuclei (3-body models). A 4-body reaction model that can be reliably applied at lower incident energies, and which takes into account the multistep intermediate-state coupling, does not exist.

Thus, the best we can do currently is to use high-energy 4-body models that rely on assumptions and approximations that break down at lower energies, and to then correct for those assumptions to extend the range of validity of the models to lower energies. For instance, non-eikonal corrections can be added to the few-body Glauber model that take into account deviations from the straight line trajectory assumption that is no longer valid below a few tens of MeV per nucleon. The most accurate method of dealing with this is to use the exact continued (EC) phase shift method [48] whereby the eikonal phase shift functions for the scattering of the projectile's constituents are replaced by the physical phase shifts.

The other high-energy assumption that is made is of course the adiabatic approximation, in which the projectile's internal Hamiltonian H_0 is replaced by its ground-state binding energy, ϵ_0. Leading-order corrections to this adiabatic limit can be evaluated by expanding the full few-body scattering wave function in powers of $H_0 + \epsilon_0$. This leads to a "non-adiabatic" correction to the leading-order adiabatic amplitude [49]. Such corrections, for elastic scattering at least, are very small even at quite low energies. Figure 4 shows a comparison between an "exact" CDCC calculation and an adiabatic (frozen halo) calculation for ^{11}Be scattering at 10 MeV/A. The reason for this remarkable agreement appears to be due to the fact that the non-adiabatic corrections tend to be concentrated at low partial waves, where absorption effects dominate. In addition, both calculations contain only a monopole Coulomb interaction and thus Coulomb breakup effects are not included.

5 Summary

We have already seen how the study of reactions with halo nuclei has driven a rapid development in few-body reaction methods over the past ten years. In the near future a large amount of experimental data will become available at lower energies as a number of ISOL facilities come on line. The challenge to theory is to develop reliable models in this regime where nuclear/Coulomb interference effects are more important, and where higher-order coupling effects must also be taken into account. It has already been shown that one-step DWBA methods are inadequate for many reactions with halo nuclei at these lower energies and the onus is therefore on extending few-body models (particularly for two-neutron halos) to describe the many processes of interest at these energies, such as fusion and transfer reactions.

More importantly, it has now been established that, even at the higher (fragmentation) energies, the nuclear-structure few-body degrees of freedom must remain entangled within the reaction formalism. This is due to the importance of higher-order coupling effects arising from the clusterised nature of the wave functions of halo nuclei.

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